Physics II
ISI B.Math
Mid Semestral Exam: September 12, 2017

Total Marks: 80
Time: 3 hours
Answer all questions

1. **(Marks : $8 \times 2 = 16$)**

For the following multiple choice questions indicate your answers by the appropriate letters (a), (b), (c) or (d).There is only one correct answer.

i) Which of the following statements is FALSE ?

(a) The internal energy of a fixed mass of gas obeying van-der Waals equation remains unchanged when it undergoes adiabatic free expansion.
(b) The entropy of a system always increases when it undergoes an irreversible transformation.
(c) $dQ = TdS$ is only true for an infinitesimal, quasistatic ,reversible transformation
(d) C_p is always greater than C_v.

(ii) Which of the following statements referring to an ideal gas is FALSE?

(a) The enthalpy of an ideal gas is independent of its volume.
(b) The temperature of a fixed mass of ideal gas decreases when it undergoes quasistatic adiabatic expansion
(c) The temperature of a fixed mass of ideal gas remains unchanged when it undergoes adiabatic free expansion
(d) The entropy of a fixed mass of ideal gas remains unchanged when it undergoes adiabatic free expansion.

(iii) A system is changed from an initial equilibrium state to the same final equilibrium state by two different processes - one reversible, and one irreversible. Which of the following is true, where ΔS refers to the change of entropy of the system?

(a) $\Delta S_{irr} = \Delta S_{rev}$
(b) $\Delta S_{irr} > \Delta S_{rev}$
(c) $\Delta S_{irr} < \Delta S_{rev}$
(d) No decision is possible with respect to (a), (b) or (c).

(iv) Experiments were conducted by four different groups to measure the variation of the heat capacity at constant volume of a solid sample with temperature in the very low temperature regime. The results obtained are given in (a),(b), (c) and (d). Which of these results is compatible with the laws of thermodynamics?

(a) $C_V = \frac{a}{\sqrt{T}}$, where a is a constant.
(b) $C_V = \frac{3}{2}R$
(c) $C_V = aT^3$, where a is a constant.

(d) $C_V = a \ln T$ where a is a constant.

v) An exact differential expression relating thermodynamic variables is given by

$$dB = CdE - FdG + HdJ$$

Which of the following would not be a new thermodynamic potential function consistent with the above expression?
(a) $B - CE$
(b) $B - HJ$
(c) $B - FG - CE$
(d) $B - HJ + FG - CE$.

vi) Two Carnot engines C_1 and C_2 operate between the temperatures T_l and T_h where $T_l < T_h$. The working material for C_1 is photon gas, with equation of state $P = \frac{1}{3}\sigma T^4$ where σ is a positive constant . The working material for C_2 is an ideal gas with usual equation of state.η_1 and η_2 are the efficiencies of engines 1 and 2 respectively. Then,
(a) $\eta_1 > \eta_2$
(b)$\eta_1 < \eta_2$
(c) $\eta_1 = \eta_2$
(d) It is not possible to conclude (a), (b) or (c) from the data given.

vii) An ideal monoatomic gas undergoes a transformation from an initial state (P_i, V_i) to a final state (P_f, V_f) where $Vf > V_i$.
(a)The work done by the gas is strictly negative
(b)The work done by the gas is strictly positive
(c) The work done by the gas is strictly zero
(c) It is not possible to conclude (b) or (c) from the information given

viii) For a Joule-Thomson throttling process when a fixed mass of ideal gas at temperature T_i and pressure P_i is forced through a porous plug to a final state with temperature T_f and pressure P_f such that $P_f < P_i$

(a) $T_f > T_i$
(b) $T_f = T_i$
(c) $T_f < T_i$
(d) It is not possible to conclude whether (a), (b) or (c) is true without knowing whether T_i is greater,less or equal to the maximum inversion temperature.

2. **(Marks: 4 + 4 + 8 = 16)**

(i) Show that it is impossible for two reversible adiabatics to intersect.(*Hint*: Assume that they do intersect, then complete the cycle with an isothermal. Show that the performance of this cycle violates the second law)
(ii) Show that an isochoric (constant volume) curve plotted on a $T - S$ diagram has a greater slope than an isobaric (constant pressure) curve at the same temperature.
(iii) A room air conditioner operates as a Carnot cycle refrigerator between an outside temperature T_h and a room at lower temperature T_i. The room gains heat from the outside at the rate $A(T_h - T_i)$;

this heat is removed by the air conditioner. The power supplied to the cooling unit is P. Show that the steady state temperature of the room is

$$T_i = (T_h + P/2A) - [(T_h + P/2A)^2 - {T_h}^2]^{\frac{1}{2}}$$

3. **(Marks: 5 + 4 + 4 + 3 = 16)**

(i) Use the first law and the fact that dS is an exact differential to show that the internal energy of an ideal gas is independent of volume.
(ii) n moles of an ideal monatomic gas in a cylinder fitted with a piston is expanded quasistatically and isothermically from volume V to volume $2V$ by keeping it in contact with a reservoir at temperature T .
(a) Find the change in entropy of the gas.
(iii) n moles of the same gas is then kept in an adiabatically insulated chamber of volume V at temperature T. It is then allowed to freely and adiabatically expand into a larger chamber of volume $2V$ by removing a partition. (b) Find the change in entropy of the gas.
(c) Find the change in entropy of the universe for cases (ii) and (iii).

4. **(Marks: 8 + 8 = 16)**

(i) Derive the equation

$$\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 P}{\partial T^2}\right)_V$$

and show that C_V of an ideal gas is a function of T only

(ii) The Joule Thomson coefficient $\mu = \left(\frac{\partial T}{\partial P}\right)_H$ is a measure of the temperature change during a throttling process for which the enthalpy $H = U + PV$ is constant. A similar measure of temperature change produced by an isentropic change of pressure is provided by the coefficient μ_S, where $\mu_S = \left(\frac{\partial T}{\partial P}\right)_S$
Show that

$$\mu_S - \mu = \frac{V}{C_P}$$

5. **(Marks: 8 + 4 + 4 =16)**

An ideal monatomic gas is taken around the following rectangular cycle on a $P - V$ diagram. The gas at a pressure $2P_0$ and volume V_0 is expanded isobarically to volume $3V_0$. The pressure is then decreased from $2P_0$ to P_0 at constant volume $3V_0$. Following this the gas is compressed isobarically at P_0 to a volume V_0. Finally the pressure of the gas is increased from P_0 to $2P_0$ at the constant volume V_0 to complete the cycle. Let this operate as a heat engine to convert the heat added to mechanical work.
(i) Evaluate the efficiency of the engine
(ii) Calculate the efficiency of an "ideal" engine operating between the same temperature extremes.
(iii) Sketch the above $P - V$ cycle on a $T - S$ plot.

Information you may (or may not) need

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$$

$$\left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$